

Fluctuation Electrodynamics – foundations

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thanks to:

Martin Wilkens, Humboldt foundation, DFG



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Fluctuation electrodynamics

what?

Basic concepts

how?

Simple examples

why not?

Beyond typical approximations

got it?

Suggested problems



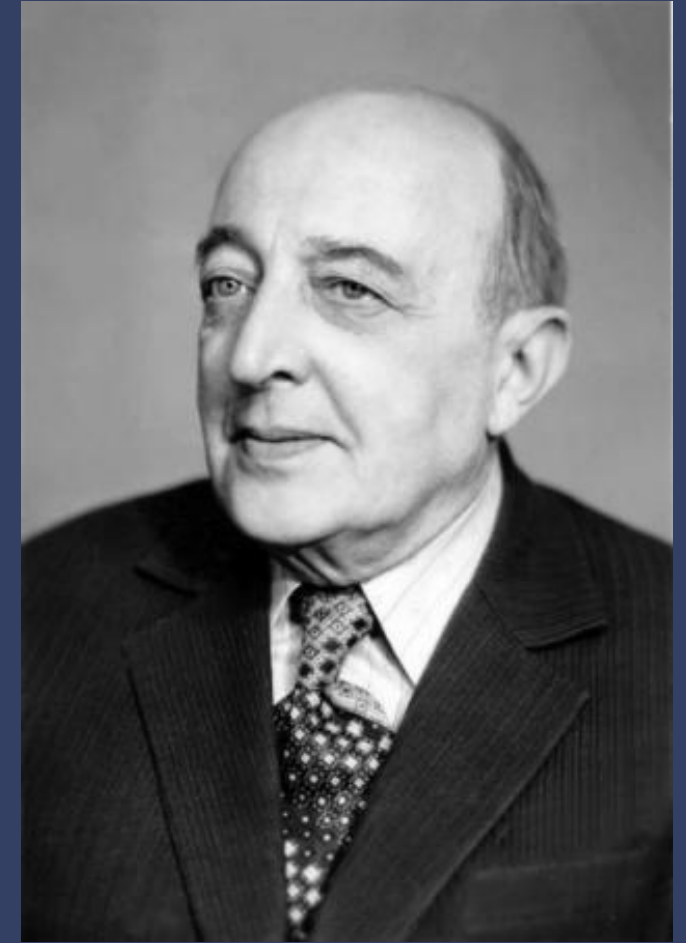
en.wikipedia.org

(1831–79)



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(1872–1946)



ru.wikipedia.org

Sergei Michailovich Rytov
(1908–96)

Fluctuation

electrodynamics

– Maxwell equations ~ 1861

$$dF = 0 \quad d^*F = j$$

– Langevin equations $\sim 1905/08$

$$m \frac{dv}{dt} + \Gamma v = F(t)$$

Langevin force $F(t) \rightarrow \begin{cases} \text{diffusion} \\ \text{thermal equilibrium} \end{cases}$

Rytov equations $\nabla \times \mathbf{H} - \partial_t \mathbf{D} = \mathbf{j}(\mathbf{x}, t) \leq 1953$

Rytov-Langevin current $\mathbf{j}(\mathbf{x}, t) \rightarrow \text{thermal equilibrium}$

- local equilibrium $T(\mathbf{x})$

Nano-scale

radiative

heat transfer

atoms \ll nano \ll λ

300 K : $\lambda \sim 10 \dots 50 \mu\text{m}$

matter \sim macroscopic continuum

fluctuat'n e'dyn

OK

thermodynamics, T , entropy S ,

heat, (ir)reversible processes

non-equilibrium:

driven by external forces ∇T , \mathbf{E}_L , $\mathbf{v}_2 - \mathbf{v}_1$

transfer of momentum = force, torque

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This lecture

- splash into Maxwell–Langevin equations
- examples: damped oscillator
hot nano-particle
heat transfer
- “quantum” remarks

→ *suggested problems*

Maxwell & Langevin

macroscopic electrodynamics

$$\begin{aligned}\partial_t \mathbf{B} &= -\nabla \times \mathbf{E} & \nabla \cdot \mathbf{B} &= 0 \\ \partial_t \mathbf{D} &= \nabla \times \mathbf{H} - \mathbf{j} & \nabla \cdot \mathbf{D} &= \rho\end{aligned}$$

linear response of (bulk) material

$$\mathbf{D} = \varepsilon_0 \varepsilon(\mathbf{x}, \omega) \mathbf{E}$$

$$\mathbf{H} = \mu_0^{-1} \mu^{-1}(\mathbf{x}, \omega) \mathbf{B}$$

... always matter that provides nonlinearity

$\varepsilon(\omega)$, $\mu(\omega)$ must be complex (!)

$\varepsilon(\mathbf{x})$, $\mu(\mathbf{x})$ cannot be local (!)

Rytov & Langevin: losses come with fluctuating sources ('forces')

$$\mathbf{j} = \mathbf{j}_{\text{free}} + \partial_t \mathbf{P}(\mathbf{x}, t) + \nabla \times \mathbf{M}(\mathbf{x}, t)$$

$$\rho = \rho_{\text{free}} - \nabla \cdot \mathbf{P}(\mathbf{x}, t)$$

cf. $m \frac{dv}{dt} = -\nabla U - \Gamma v + F(t)$

Maxwell & Langevin

macroscopic electrodynamics

$$-i\frac{\omega}{c^2}\varepsilon(\mathbf{x},\omega)\mathbf{E} = \nabla \times \mu^{-1}(\mathbf{x},\omega)\mathbf{B} - \mu_0\mathbf{j}(\mathbf{x},\omega) \qquad -i\omega\mathbf{B} = \nabla \times \mathbf{E}$$

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Maxwell-Langevin equation: 'stochastic differential equation'

= physicists' code $0 = \langle \mathbf{P}(\mathbf{x}, t) \rangle$

word for averages

$$0 \neq \langle \mathbf{P}(\mathbf{x}, t)\mathbf{P}(\mathbf{x}', t') \rangle = \int \frac{d\omega}{2\pi} S_P(\mathbf{x}, \mathbf{x}', \omega) e^{i\omega(t-t')} \qquad \text{spectral density}$$

Maxwell & Langevin

macroscopic electrodynamics

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Rytov:

$$S_{P,ij}(\mathbf{x}, \mathbf{x}', \omega) \approx 2\hbar\bar{N}(\omega)\varepsilon_0 \text{Im} \varepsilon_{ij}(\mathbf{x}, \omega)\delta(\mathbf{x} - \mathbf{x}')$$

... fluctuation-dissipation relation

$$\text{Bose-Einstein distribution } \bar{N}(\omega) = \frac{1}{\exp(\hbar\omega/k_B T) - 1} \approx \frac{k_B T}{\hbar\omega}$$

Maxwell & Langevin

Suggested problems

(1) Energy balance in lossy medium

water IR absorption $\alpha \sim 10^3 \text{ cm}^{-1}$

field $E = 1 \text{ mV}/\mu\text{m}$: energy density $\sim 0.03 \text{ meV}/\mu\text{m}^3$

absorbed power density $\sim 5 \text{ meV}/\text{ps}/\mu\text{m}^3$

time scale for losses $c\alpha \sim \omega/2\pi$ frequency

(2) Temperature: Bjerrum length $\frac{e^2}{\epsilon_0 k_B T} = 0.7 \mu\text{m}$

(3) Fluctuating dipole moment in volume ΔV : $\mathbf{d}(t) = \int_{\Delta V} dx \mathbf{P}(\mathbf{x}, t)$

estimate polarization energy $\frac{\Delta V}{\epsilon_0} \frac{\langle d^2 \rangle_\omega \Delta\omega}{(\Delta V)^2}$

(4) Count degrees of freedom in volume $1 \mu\text{m}^3$:

photon modes & matter (phonon) modes (bandwidth $\Delta\omega$)

Maxwell & Langevin

Remarks

- equilibrium: fluctuation–dissipation (FD) relation for (quantum) fields Callen & Welton 1951
- FD relations (theorem)
 - Einstein (1905), Langevin (1908): diffusion $D \leftrightarrow \Gamma$ friction
 - detailed balance: up/down rate $\sim e^{-\Delta E/k_B T}$ Boltzmann
 - Kubo-Martin-Schwinger (KMS) relation $\langle A(t)B(t') \rangle_T$ vs $\langle B(t')A(t) \rangle_T$

Maxwell & Langevin

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Einstein (1905), Langevin (1908): diffusion $D = k_B T \Gamma$ friction
detailed balance: up/down rate $\sim e^{-\Delta E/k_B T}$ Boltzmann
Kubo-Martin-Schwinger (KMS) relation $S_{AB}(\omega) = e^{-\hbar\omega/k_B T} S_{BA}(\omega)$
- Rytov in full power: non-equilibrium
local temperature $S_P(\mathbf{x}, \omega) \sim \bar{N}(\omega, T(\mathbf{x})) \text{Im} \varepsilon(\mathbf{x}, \omega)$ Lifshitz & Pitaevskii 1980s
- local equilibrium: on sub- λ scale, matter thermalizes ‘fast enough’
‘photons are a poor thermostat’ Planck 1900 / PLANCK 2013
- local approximation $\varepsilon(\mathbf{x}, \omega)$ for dielectric response
non-local media (plasma/screening, effective medium/unit cell)
genuine surface response $P(x, y, \omega)\delta(z) \sim \chi(\omega)E(x, y, z = 0, \omega)$

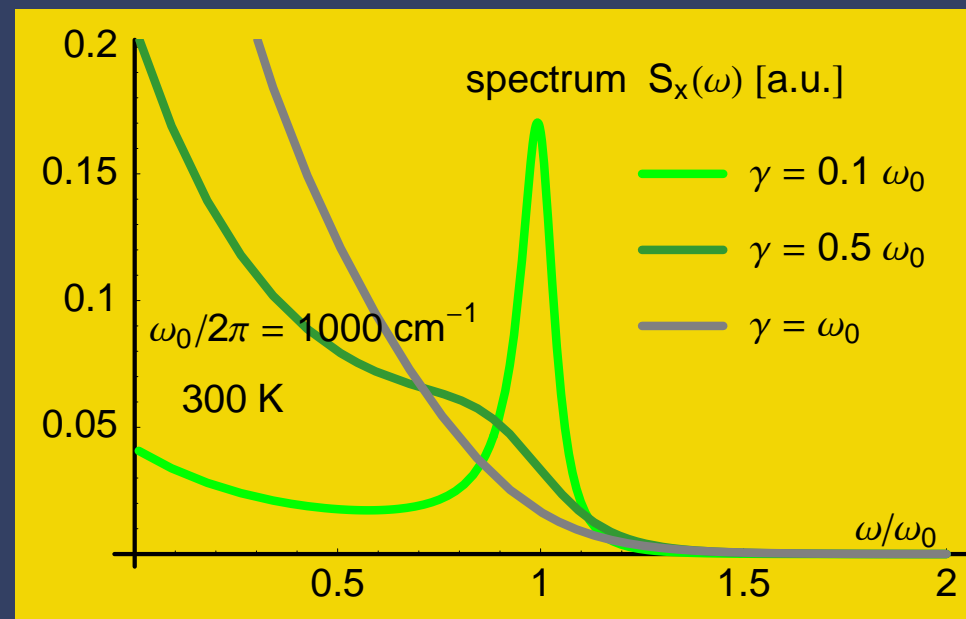
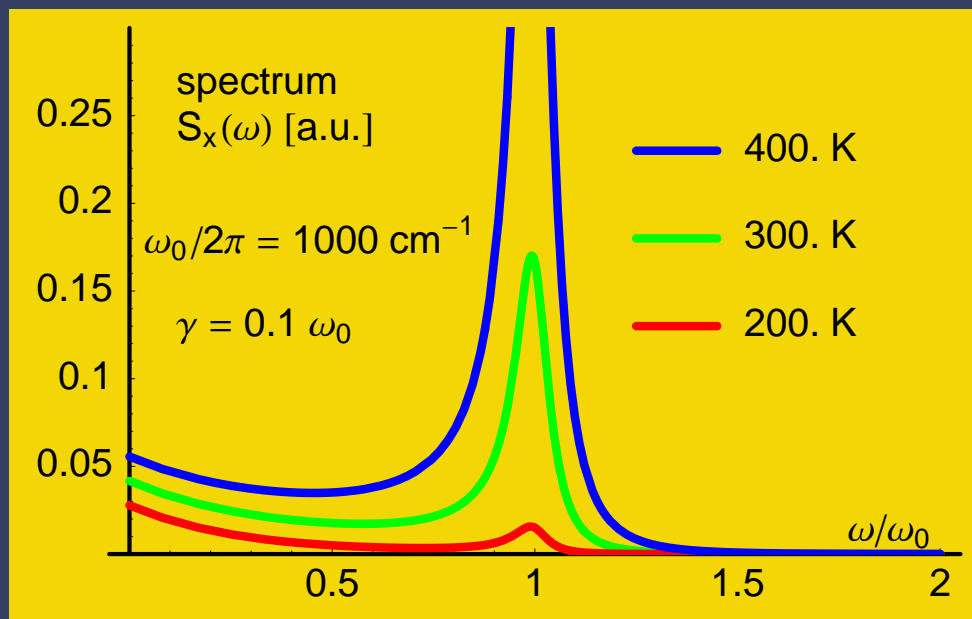
Examples

Damped harmonic oscillator $m\ddot{x} + \Gamma\dot{x} + Kx = F(t)$

Langevin force spectrum $S_F(\omega) = 2\hbar\bar{N}(\omega)\omega \operatorname{Re} \Gamma(\omega)$ white vs colored (discussion)

Oscillator spectrum (equilibrium, long-time limit)

$$S_x(\omega) = \frac{S_F(\omega)}{|-m\omega^2 - i\omega\Gamma(\omega) + K|^2} = 2\hbar\bar{N}(\omega) \operatorname{Im} \left(\frac{1}{-m\omega^2 - i\omega\Gamma(\omega) + K} \right)$$



non-equilibrium: relaxation dynamics, two-temperature driving ...

quantum questions: zero-point energy, negative frequencies, overdamped limit ...

Examples

Hot nano-sphere



spectrum of dipole moment

$$S_d(\omega) \approx 2\hbar\bar{N}(\omega)4\pi a^3 \epsilon_0 \operatorname{Im} \frac{\epsilon(\omega) - 1}{\epsilon(\omega) + 2}$$

polarizability $\operatorname{Im} \alpha(\omega)$

• emitted field

$$\mathbf{E}(\mathbf{r}, \omega) = \frac{\omega^2 e^{i\omega r/c}}{4\pi\epsilon_0 c^2 r} \left\{ (\mathbf{d} - 3\hat{\mathbf{r}}(\hat{\mathbf{r}} \cdot \mathbf{d})) \left(\frac{-1}{(\omega r/c)^2} + \frac{i}{\omega r/c} \right) + \mathbf{d} - \hat{\mathbf{r}}(\hat{\mathbf{r}} \cdot \mathbf{d}) \right\}$$

$$\langle \mathbf{E}(\mathbf{r}, \omega) \rangle = 0 = \mathbf{G}(\mathbf{r}, \mathbf{0}, \omega) \cdot \mathbf{d}(\omega)$$

Green function/tensor/dyadic
→ this week

Examples

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Green function/tensor/dyadic
→ this week

Suggested problems

average intensity “ $\langle |\mathbf{E}(\mathbf{r}, \omega)|^2 \rangle$ ” vs distance r

average Poynting vector “ $\operatorname{Re} \langle \mathbf{E}^*(\mathbf{r}, \omega) \times \mathbf{H}(\mathbf{r}, \omega) \rangle$ ”

total emitted power & cooling rate/time

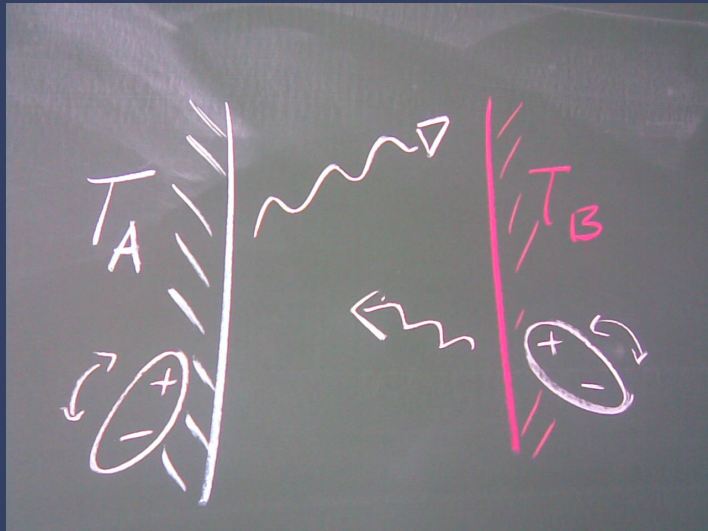
array of nano-particles

→ P Ben-Abdallah

- core technique for non-equilibrium Rytov theory
= ‘incoherent summation’ over elementary source volumes

Examples

Radiative heat transfer



Poynting vector $\text{Re} \langle \mathbf{E}^*(\mathbf{r}, \omega) \times \mathbf{H}(\mathbf{r}, \omega) \rangle_z$ T_{z0} current

= (left sources) + (right sources)

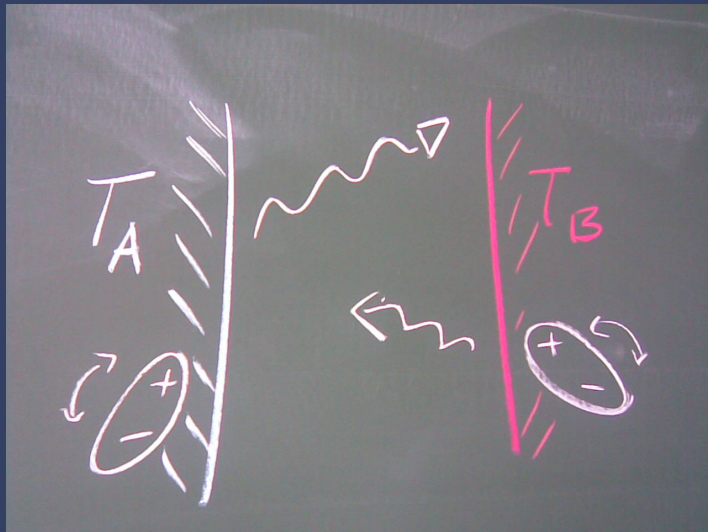
$\approx (T_B - T_A) \times h(\text{distance}, \bar{T})$

→ this week's programme

- Casimir interaction = momentum transfer, stress tensor $\langle T_{ij} \rangle$

Examples

Radiative heat transfer



Poynting vector $\text{Re} \langle \mathbf{E}^*(\mathbf{r}, \omega) \times \mathbf{H}(\mathbf{r}, \omega) \rangle_z$ T_{z0} current

= (left sources) + (right sources)

$\approx (T_B - T_A) \times h(\text{distance}, \bar{T})$

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Typical / tacit assumptions

- homogeneous temperature profile *suggested problem* conduction faster than radiation
- Langevin sources not correlated radiative coupling slower than local relaxation
- no heat extracted from vacuum fluctuations but: unstable vacua

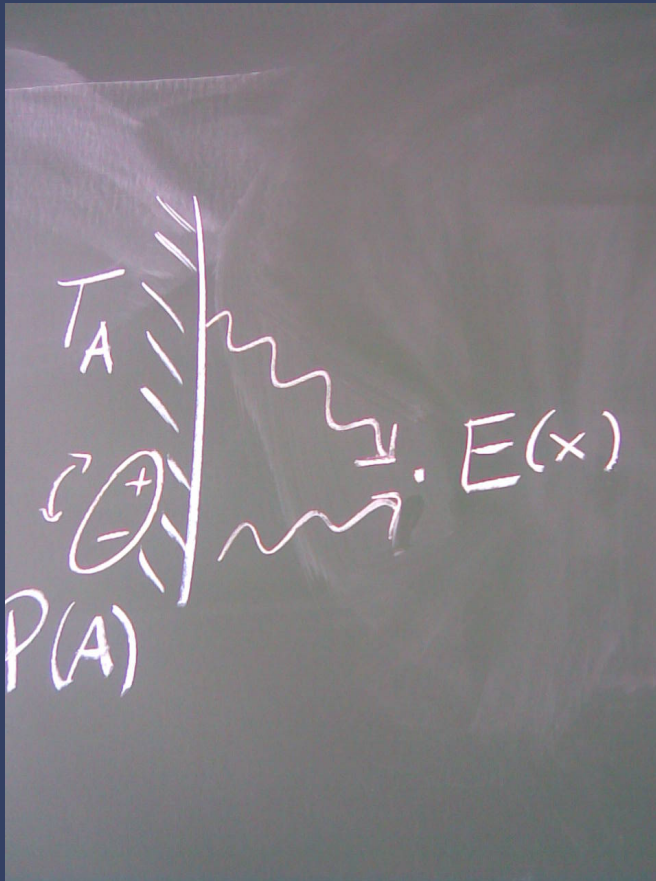
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Examples

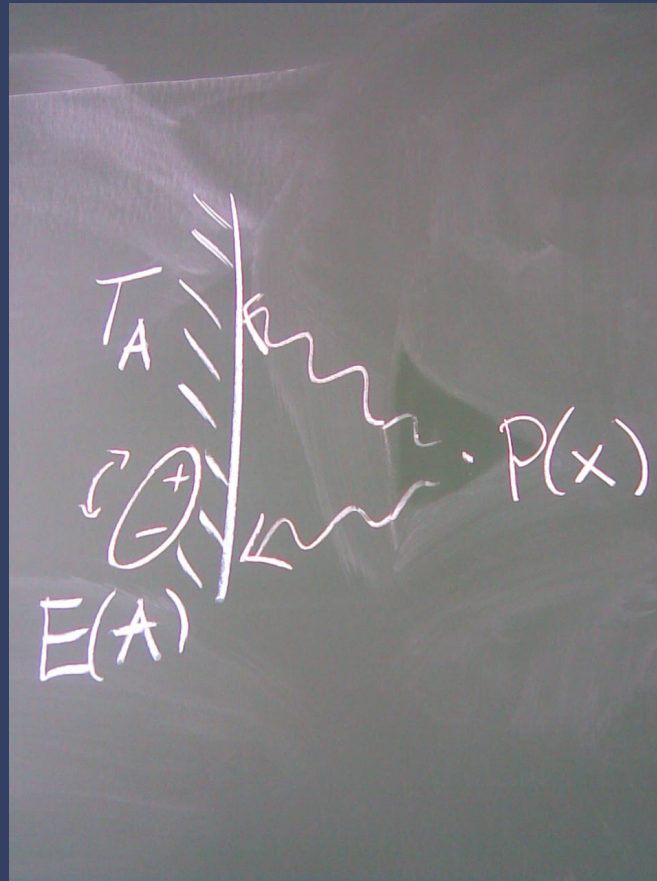
Radiative heat transfer

- convenient shortcut: generalized Kirchhoff law

Dorofeyev & Vinogradov (*Phys Rep* 2011)

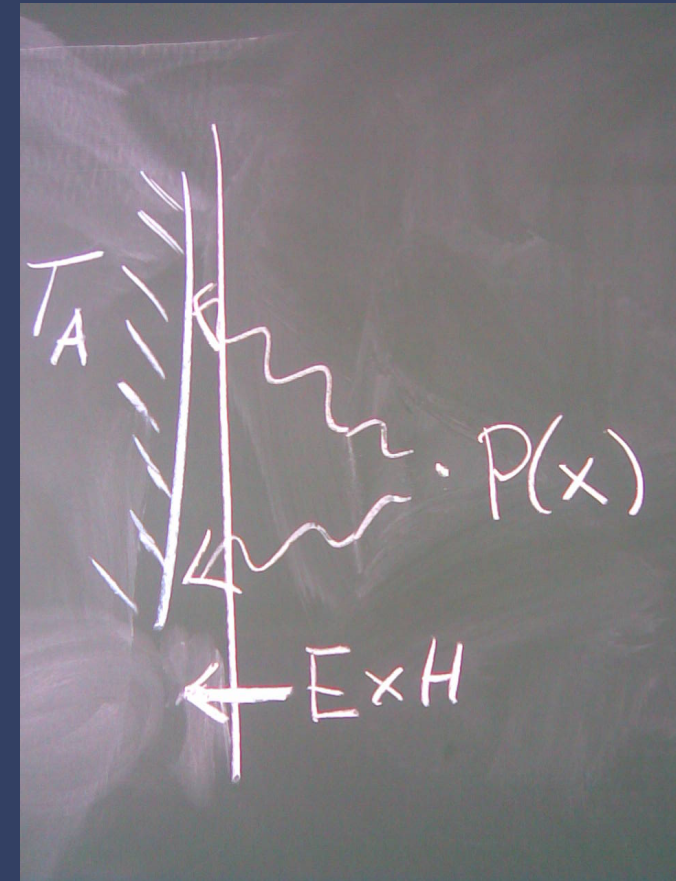


emission by sources



reciprocity

$$G(\mathbf{x}, \mathbf{x}') = G^T(\mathbf{x}', \mathbf{x})$$



Poynting vector

“Quantum” remarks

(1) ‘first quantization’ QM electrodynamics

Heisenberg $\Delta p \Delta x \sim \hbar$ (Fourier)

Planck & Einstein $\hbar\omega$ $\epsilon \mathbf{E}^2 + \mu \mathbf{H}^2$

de Broglie $\hbar \mathbf{k}$ $\epsilon \mathbf{E} \times \mathbf{B}$

(2) ‘second quantization’

Pauli & Jordan $\Delta E \Delta A \sim \hbar$

$$i [A_i(\mathbf{x}), E_j(\mathbf{x}')] = \frac{\hbar}{\epsilon_0} \delta_{ij}^\perp(\mathbf{x} - \mathbf{x}')$$

$$i [B_i(\mathbf{x}), E_j(\mathbf{x}')] = -\frac{\hbar}{\epsilon_0} \epsilon_{ijk} \partial_k \delta(\mathbf{x} - \mathbf{x}')$$

Ordering of operators

Feynman $\langle T\{E(t)E(t')\} \rangle$ $D_E(\omega)$ even

Green $i\langle [E(t), E(t')] \rangle \Theta(t - t')$ $\text{Im } G(\omega)$ odd

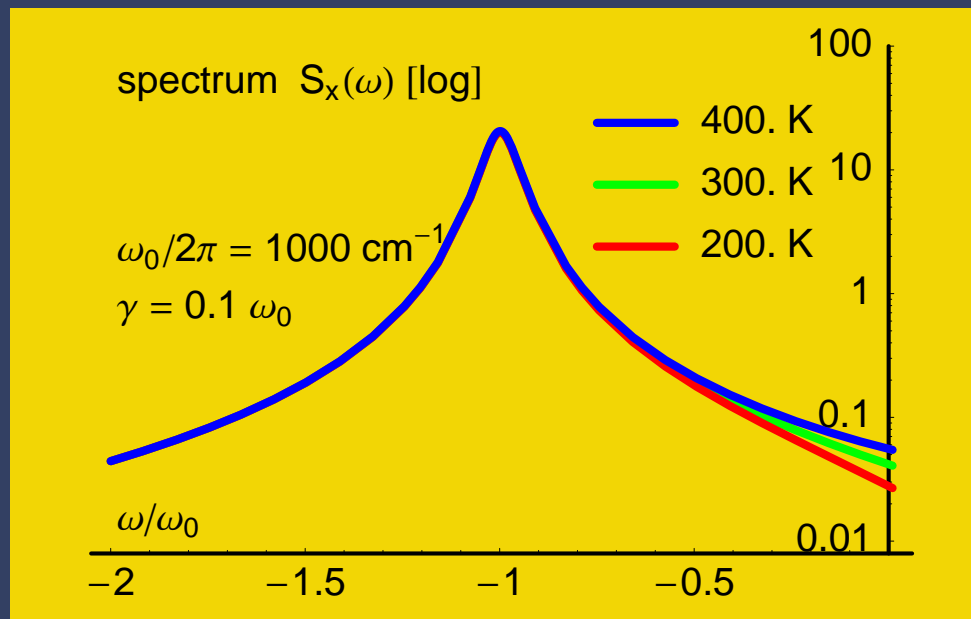
Hadamard $\langle \{E(t), E(t')\} \rangle$ $H_E(\omega)$ real and even $\sim \bar{N}(\omega) + \frac{1}{2}$

Glauber (?) $\langle E(t)E(t') \rangle$ $S_E(\omega)$ positive $\sim \bar{N}(\omega), \quad \omega > 0$
 $e^{i\omega(t-t')}$

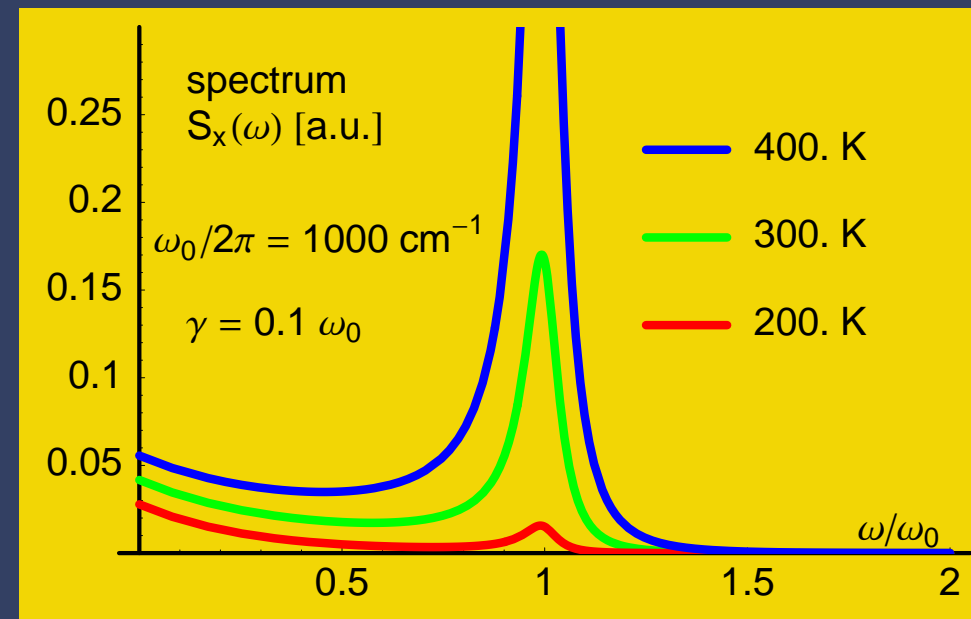
“Quantum” remarks

Damped oscillator in equilibrium

Glauber $S_x(\omega)$ at negative ...



... and positive frequencies

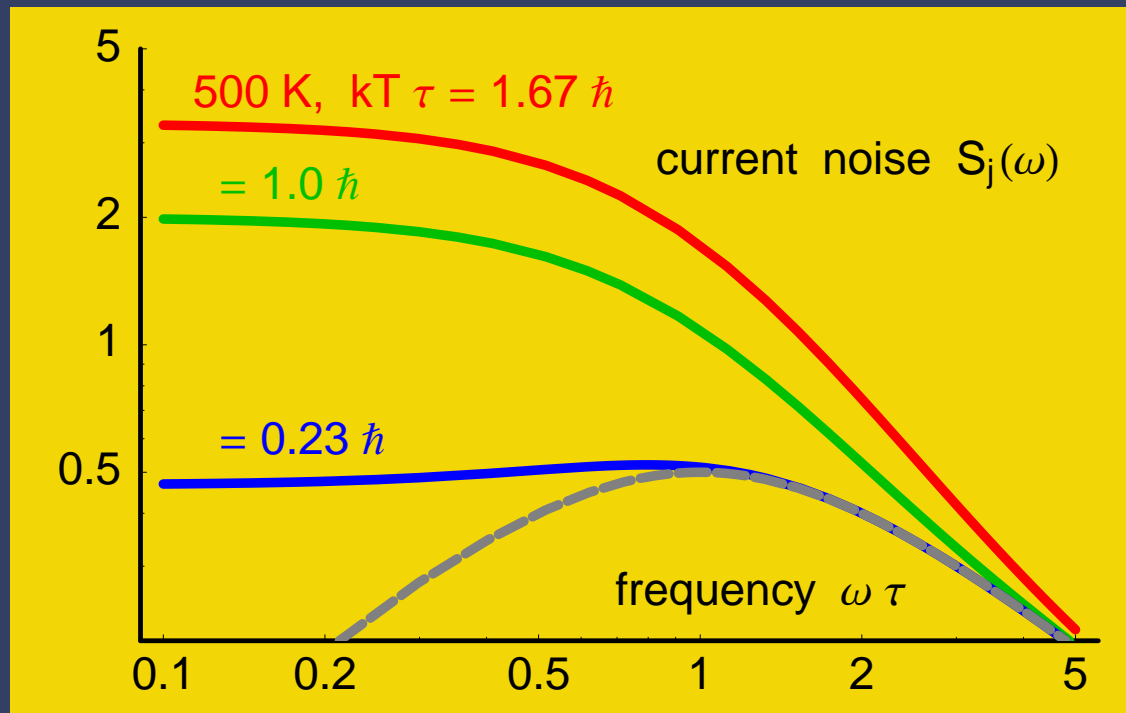


quantum noise / zero-point fluctuations

“Quantum” remarks

Electric current noise in metal

$$\text{symmetrized } H_j(\omega) \sim (\bar{N}(\omega) + \frac{1}{2}) \omega \text{Re } \sigma(\omega)$$



quantum noise of overdamped/diffusive field

- Casimir interaction: see Intravaia & CH (*Phys Rev Lett* 2009)

“Quantum” remarks

Suggested problems

Glauber spectrum: $-\bar{N}(-\omega) = \bar{N}(\omega) + 1$ asymmetric, still positive

Magnetic flux quantization? usually need electron charge for flux quantum, so it's beyond field quantization? (cf. magnetic monopole)

Why do annihilation operators evolve with positive frequencies?

$$a(t) = a(0) \exp(-i\omega t)$$

Can all observers agree on the sign of a frequency? (not always!)

anomalous Doppler effect (Ginzburg, review: *Phys Uspekhi* 1996):

$$\omega' = \gamma(\omega + \mathbf{v} \cdot \mathbf{k}) < 0$$

quantum friction, pair creation, Cherenkov radiation

Summary

stochastic electrodynamics Rytov = Maxwell + Langevin

route towards thermal equilibrium

QED: preserve commutators

separation of time scales

field absorption vs heat transport

Examples

stochastic field correlations: linear in sources

natural non-equilibrium technique (local vs global eq)

'macroscopic' response functions sufficient, even for quantum noise

(ϵ , μ , surface response ...)

Glimpses ... of the land beyond Rytov

Suggested problem

wikipedia entries about S. M. Rytov
(english, français, deutsch ...)

Thanks to the group!

Francesco Intravaia (→ Nottingham)	Casimir physics	<i>Phys Rev A</i> 2010
Jürgen Schiefele (→ Madrid)	atom chip & BEC	<i>Phys Lett A</i> 2011
Harald Haakh (→ Erlangen)	superconducting atom chips	<i>Eur Phys J B</i> 2012
Geesche Boedecker (PhD)	non-Markovian QED	<i>Ann Phys (Berlin)</i> 2012
Giuseppe Cammarata (post doc)	quantization of coupled oscillators	
Alexander Kegeles (PhD)	entanglement production	
Gregor Pieplow (Dipl)	quantum friction	<i>New J Phys</i> 2013
Ralf Saplata (Dipl)	failures of adiabaticity	
Abdoulaye Diallo (Dipl)	border of BEC	
Timo Felbinger	Illarion Dorofeyev (Nizhny Novgorod)	
Martin Wilkens	Vanik E. Mkrtchian (Ashtarak)	

Appendix – forgotten references

- von Laue: Thermal radiation in absorbing bodies (Ann. Phys. (Leipzig) 1910)
- Bakker & Heller: 'Quantum' Brownian motion in electric resistances (Physica 1939)
- Jauch & Watson: Phenomenological Quantum-Electrodynamics (Phys Rev 1948)
- Callen & Welton: Irreversibility and generalized noise (Phys Rev 1951)
- De Groot & al: Series on relativistic thermodynamics (Physica \geq 1953)
- V. L. Ginzburg: Electrical fluctuations (Fortschr Phys 1953)
- Linder: Thermal Van der Waals interactions (J Chem Phys \geq 1960)
- van Kampen: FD relation in non-linear systems? (Physica 1960) vs Polevoi & Rytov (Theor Math Phys 1975)
- Morris & Fürth: Spatial field correlations near conducting surfaces (Physica 1960)
see also Fuchs (Radiophys Quant Electr 1965)
- Jaynes & Cummings: Quantum vs semiclassical radiation theories (Proc. IEEE 1963)
- Agarwal: FD theorems and series on field quantization (Z Phys 1972; Phys Rev A 1975)
- Boyer: Connection between Rytov and quantum electrodynamics (Phys Rev D 1975)
- Eckhart: Problems with FD relations for heat transfer (Opt Commun \geq 1982)
- Henry & Kazarinov: Quantum noise in photonics (Rev Mod Phys 1996)

Appendix — Fluctuation-dissipation relation

Cross correlation
spectrum:

$$\mathcal{E}_{ij}(\mathbf{x}, \mathbf{x}'; \omega) = \int_{-\infty}^{+\infty} d\tau e^{-i\omega\tau} \langle E_i(\mathbf{x}, t + \tau) E_j(\mathbf{x}', t) \rangle$$

Linear response:

$$\mathcal{G}_{ij}(\mathbf{x}, \mathbf{x}'; \omega) = \frac{i}{\hbar} \int_0^{+\infty} d\tau e^{i\omega\tau} \langle [E_i(\mathbf{x}, \tau), E_j(\mathbf{x}', 0)] \rangle$$

Canonical ensemble:
(KMS relation)

$$\int_{-\infty}^{+\infty} d\tau e^{i\omega\tau} \langle A(\tau) B(0) \rangle = e^{\hbar\omega/k_B T} \int_{-\infty}^{+\infty} d\tau e^{i\omega\tau} \langle B(0) A(\tau) \rangle$$

→ Fluctuation–dissipation relation:

$$\mathcal{E}_{ij}(\mathbf{x}', \mathbf{x}; \omega) = 2\hbar\bar{N}(\omega) \frac{\mathcal{G}_{ji}(\mathbf{x}', \mathbf{x}; \omega) - [\mathcal{G}_{ij}(\mathbf{x}, \mathbf{x}'; \omega)]^*}{2i}$$

Reciprocity: $\mathcal{G}_{ji}(\mathbf{x}', \mathbf{x}; \omega) = \mathcal{G}_{ij}(\mathbf{x}, \mathbf{x}'; \omega)$