Fluctuation Electrodynamics – foundations

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thanks to:

Martin Wilkens, Humboldt foundation, DFG

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www.quantum.physik.uni-potsdam.de

Fluctuation electrodynamics

what?

Basic concepts

how?

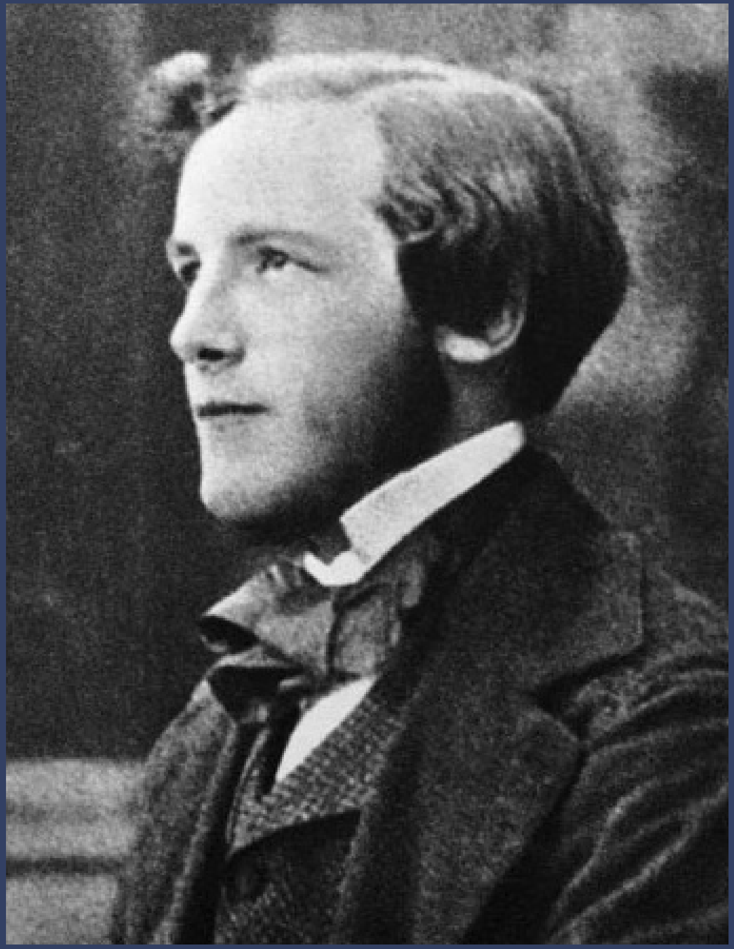
Simple examples

why not?

Beyond typical approximations

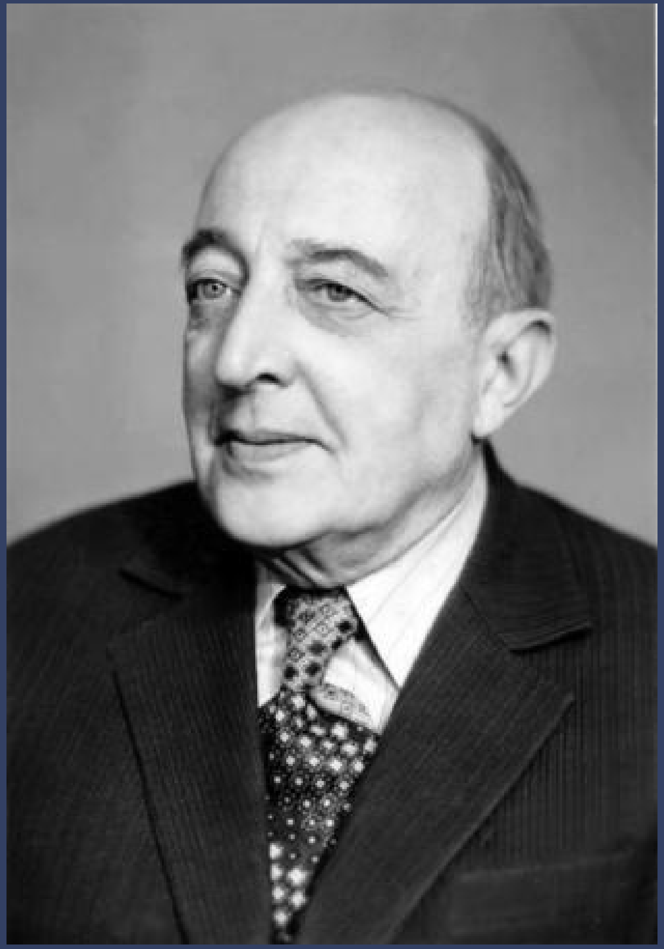
got it?

Suggested problems



(1831–79)

(1872–1946)



Sergei Michailovich Rytov

(1908–96)

Fluctuation electrodynamics

– Maxwell equations ~ 1861

$$\mathrm{d}F = 0 \qquad \mathrm{d}\,{}^{\star}F = j$$

– Langevin equations $\sim 1905/08$

$$m\frac{dv}{dt} + \Gamma v = F(t)$$

Langevin force $F(t) \to \begin{cases} \text{diffusion} \\ \text{thermal equilibrium} \end{cases}$

$$\boxed{\text{Rytov equations} \qquad \nabla \times \mathbf{H} - \partial_t \mathbf{D} = \mathbf{j}(\mathbf{x}, t)} \leq 1953$$

Rytov-Langevin current $\mathbf{j}(\mathbf{x}, t) \to$ thermal equilibrium

- local equilibrium $T(\mathbf{x})$

Nano-scale radiative heat transfer

fluctuat'n e'dyn

OK

atoms $\ll$ nano $\ll \lambda$

$300\,$K : $\lambda \sim 10 \ldots 50\,\mu$m

matter $\sim$ macroscopic continuum

thermodynamics, T, entropy S,

heat, (ir)reversible processes

non-equilibrium:

driven by external forces ∇T, $\mathbf{E}_L$, $\mathbf{v}_2 - \mathbf{v}_1$

transfer of momentum $=$ force, torque

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This lecture

- splash into Maxwell–Langevin equations
- examples: damped oscillator
  hot nano-particle
  heat transfer
- "quantum" remarks

$\rightarrow$ *suggested problems*

Maxwell & Langevin

macroscopic electrodynamics

$$\partial_t \mathbf{B} = -\nabla \times \mathbf{E} \qquad \nabla \cdot \mathbf{B} = 0$$

$$\partial_t \mathbf{D} = \nabla \times \mathbf{H} - \mathbf{j} \qquad \nabla \cdot \mathbf{D} = \rho$$

linear response of (bulk) material

. . . always matter that provides nonlinearity

$$\mathbf{D} = \varepsilon_0 \varepsilon(\mathbf{x}, \omega) \mathbf{E}$$

$$\mathbf{H} = \mu_0^{-1} \mu^{-1}(\mathbf{x}, \omega) \mathbf{B}$$

$\varepsilon(\omega)$, $\mu(\omega)$ must be complex (!)

$\varepsilon(\mathbf{x})$, $\mu(\mathbf{x})$ cannot be local (!)

Rytov & Langevin: losses come with fluctuating sources ('forces')

$$\mathbf{j} = \mathbf{j}_{\text{free}} + \partial_t \mathbf{P}(\mathbf{x}, t) + \nabla \times \mathbf{M}(\mathbf{x}, t)$$

$$\rho = \rho_{\text{free}} - \nabla \cdot \mathbf{P}(\mathbf{x}, t)$$

cf. $$m \frac{dv}{dt} = -\nabla U - \Gamma v + F(t)$$

Maxwell & Langevin

macroscopic electrodynamics

$$-\mathrm{i}\frac{\omega}{c^2}\varepsilon(\mathbf{x},\omega)\mathbf{E} \quad = \quad \nabla\times\mu^{-1}(\mathbf{x},\omega)\mathbf{B} - \mu_0\mathbf{j}(\mathbf{x},\omega) \qquad\qquad -\mathrm{i}\omega\mathbf{B} = \nabla\times\mathbf{E}$$

Rytov & Langevin: losses come with fluctuating sources ('forces')

$$\mathbf{j} \quad = \quad \mathbf{j}_{\mathrm{free}} - \mathrm{i}\omega\mathbf{P}(\mathbf{x},\omega) + \nabla\times\mathbf{M}(\mathbf{x},\omega)$$

$$\rho \quad = \quad \rho_{\mathrm{free}} - \nabla\cdot\mathbf{P}(\mathbf{x},\omega)$$

Maxwell-Langevin equation: 'stochastic differential equation'

= physicists' code word for averages

$$0 \quad = \quad \langle\mathbf{P}(\mathbf{x},t)\rangle$$

$$0 \quad \neq \quad \langle\mathbf{P}(\mathbf{x},t)\mathbf{P}(\mathbf{x}',t')\rangle = \int\frac{\mathrm{d}\omega}{2\pi}S_P(\mathbf{x},\mathbf{x}',\omega)\,\mathrm{e}^{\mathrm{i}\omega(t-t')}$$

spectral density

Maxwell & Langevin

macroscopic electrodynamics

$$-\mathrm{i}\frac{\omega}{c^2}\varepsilon(\mathbf{x},\omega)\mathbf{E} = \nabla\times\mu^{-1}(\mathbf{x},\omega)\mathbf{B} - \mu_0\mathbf{j}(\mathbf{x},\omega) \qquad -\mathrm{i}\omega\mathbf{B} = \nabla\times\mathbf{E}$$

Rytov & Langevin: losses come with fluctuating sources ('forces')

$$\begin{aligned}\mathbf{j} &= \mathbf{j}_{\text{free}} - \mathrm{i}\omega\mathbf{P}(\mathbf{x},\omega) + \nabla\times\mathbf{M}(\mathbf{x},\omega)\\ \rho &= \rho_{\text{free}} - \nabla\cdot\mathbf{P}(\mathbf{x},\omega)\end{aligned}$$

Maxwell-Langevin equation: 'stochastic differential equation'

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$$\begin{aligned}0 &= \langle\mathbf{P}(\mathbf{x},t)\rangle\\ 0 &\neq \langle\mathbf{P}(\mathbf{x},t)\mathbf{P}(\mathbf{x}',t')\rangle = \int\frac{\mathrm{d}\omega}{2\pi}S_P(\mathbf{x},\mathbf{x}',\omega)\,\mathrm{e}^{\mathrm{i}\omega(t-t')}\end{aligned}$$

spectral density

Rytov: $\boxed{S_{P,ij}(\mathbf{x},\mathbf{x}',\omega) \approx 2\hbar\bar{N}(\omega)\varepsilon_0\,\mathrm{Im}\,\varepsilon_{ij}(\mathbf{x},\omega)\delta(\mathbf{x}-\mathbf{x}')}$ ... fluctuation-dissipation relation

Bose-Einstein distribution $\bar{N}(\omega) = \dfrac{1}{\exp(\hbar\omega/k_BT)-1} \approx \dfrac{k_BT}{\hbar\omega}$

Maxwell & Langevin

Suggested problems

(1) Energy balance in lossy medium

water IR absorption $\alpha \sim 10^3\,\text{cm}^{-1}$

field $E = 1\,\text{mV}/\mu\text{m}$: energy density $\sim 0.03\,\text{meV}/\mu\text{m}^3$

absorbed power density $\sim 5\,\text{meV}/\text{ps}/\mu\text{m}^3$

time scale for losses $c\alpha \sim \omega/2\pi$ frequency

(2) Temperature: Bjerrum length $\dfrac{e^2}{\varepsilon_0 k_B T} = 0.7\,\mu\text{m}$

(3) Fluctuating dipole moment in volume ΔV: $\mathbf{d}(t) = \displaystyle\int_{\Delta V} \mathrm{d}x\,\mathbf{P}(\mathbf{x}, t)$

estimate polarization energy $\dfrac{\Delta V}{\varepsilon_0} \dfrac{\langle d^2 \rangle_\omega \Delta\omega}{(\Delta V)^2}$

(4) Count degrees of freedom in volume $1\,\mu\text{m}^3$:

photon modes & matter (phonon) modes (bandwidth $\Delta\omega$)

Maxwell & Langevin

Remarks

— equilibrium: fluctuation–dissipation (FD) relation for (quantum) fields Callen & Welton 1951

— FD relations (theorem)
Einstein (1905), Langevin (1908): diffusion $D \leftrightarrow \Gamma$ friction
detailed balance: up/down rate $\sim \mathrm{e}^{-\Delta E/k_B T}$ Boltzmann
Kubo-Martin-Schwinger (KMS) relation $\langle A(t)B(t')\rangle_T$ vs $\langle B(t')A(t)\rangle_T$

Maxwell & Langevin

Remarks

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— FD relations (theorem)
Einstein (1905), Langevin (1908): diffusion $D = k_B T\,\Gamma$ friction
detailed balance: up/down rate $\sim \mathrm{e}^{-\Delta E/k_B T}$ Boltzmann
Kubo-Martin-Schwinger (KMS) relation $S_{AB}(\omega) = \mathrm{e}^{-\hbar\omega/k_B T} S_{BA}(\omega)$

— Rytov in full power: non-equilibrium
local temperature $S_P(\mathbf{x}, \omega) \sim \bar{N}(\omega, T(\mathbf{x}))\,\mathrm{Im}\,\varepsilon(\mathbf{x}, \omega)$ Lifshitz & Pitaevskii 1980s

— local equilibrium: on sub-λ scale, matter thermalizes ‘fast enough’
‘photons are a poor thermostat’   Planck 1900 / PLANCK 2013

— local approximation $\varepsilon(\mathbf{x}, \omega)$ for dielectric response
non-local media (plasma/screening, effective medium/unit cell)
genuine surface response $P(x, y, \omega)\delta(z) \sim \chi(\omega) E(x, y, z = 0, \omega)$

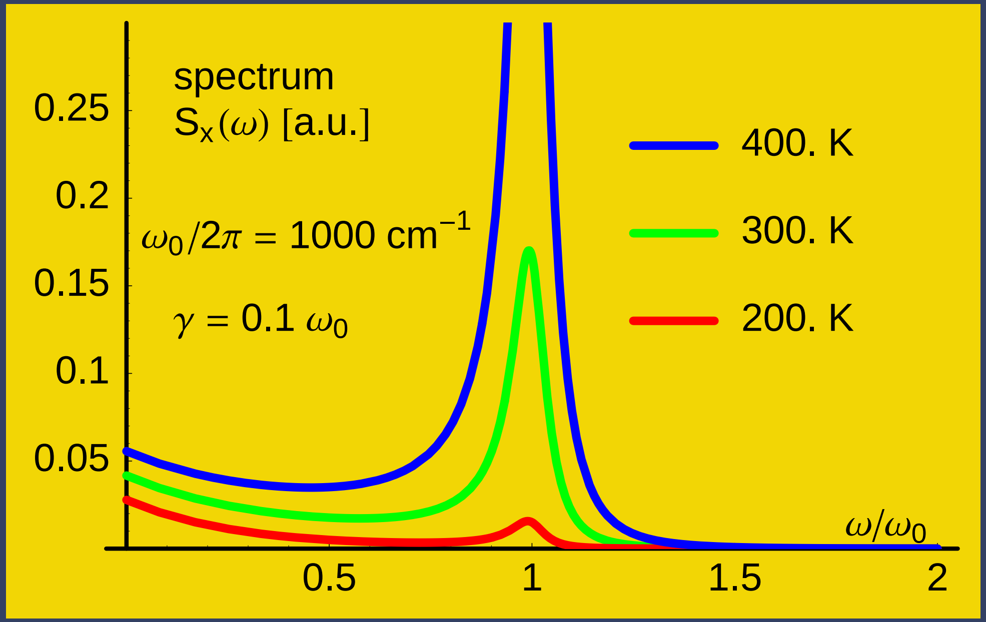
Examples

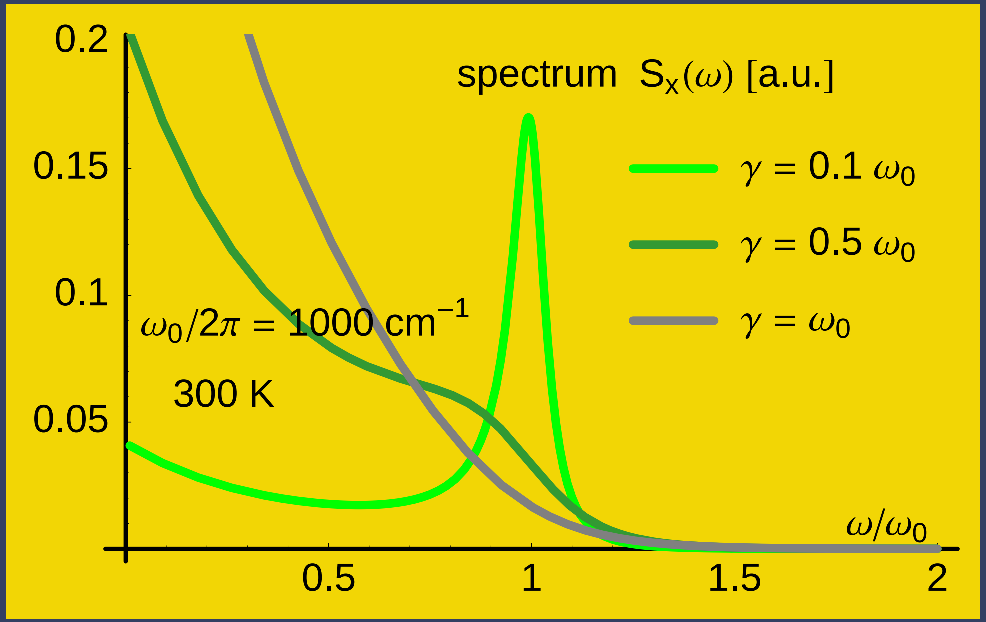
Damped harmonic oscillator $\quad m\ddot{x} + \Gamma\dot{x} + Kx = F(t)$

Langevin force spectrum $\quad S_F(\omega) = 2\hbar\bar{N}(\omega)\omega\,\mathrm{Re}\,\Gamma(\omega)$ $\quad$ white vs colored (discussion)

Oscillator spectrum (equilibrium, long-time limit)

$$S_x(\omega) = \frac{S_F(\omega)}{|-m\omega^2 - \mathrm{i}\omega\Gamma(\omega) + K|^2} = 2\hbar\bar{N}(\omega)\,\mathrm{Im}\Big(\frac{1}{-m\omega^2 - \mathrm{i}\omega\Gamma(\omega) + K}\Big)$$





non-equilibrium: relaxation dynamics, two-temperature driving . . .

quantum questions: zero-point energy, negative frequencies, overdamped limit . . .

Examples

Hot nano-sphere

spectrum of dipole moment

$$S_d(\omega) \approx 2\hbar \bar{N}(\omega) 4\pi a^3 \varepsilon_0 \,\mathrm{Im}\, \frac{\varepsilon(\omega) - 1}{\varepsilon(\omega) + 2}$$

polarizability $\mathrm{Im}\,\alpha(\omega)$

• emitted field

$$\mathbf{E}(\mathbf{r}, \omega) = \frac{\omega^2\, \mathrm{e}^{\mathrm{i}\omega r/c}}{4\pi\varepsilon_0 c^2 r} \left\{ (\mathbf{d} - 3\hat{\mathbf{r}}(\hat{\mathbf{r}} \cdot \mathbf{d})) \left(\frac{-1}{(\omega r/c)^2} + \frac{\mathrm{i}}{\omega r/c} \right) + \mathbf{d} - \hat{\mathbf{r}}(\hat{\mathbf{r}} \cdot \mathbf{d}) \right\}$$

$$\langle \mathbf{E}(\mathbf{r}, \omega) \rangle = 0 \qquad = \mathbf{G}(\mathbf{r}, \mathbf{0}, \omega) \cdot \mathbf{d}(\omega)$$

Green function/tensor/dyadic

$\rightarrow$ this week

Examples

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Green function/tensor/dyadic

$\rightarrow$ this week

Suggested problems

average intensity "$\langle |\mathbf{E}(\mathbf{r}, \omega)|^2 \rangle$" vs distance r

average Poynting vector "$\mathrm{Re}\,\langle \mathbf{E}^*(\mathbf{r}, \omega) \times \mathbf{H}(\mathbf{r}, \omega) \rangle$"

total emitted power & cooling rate/time

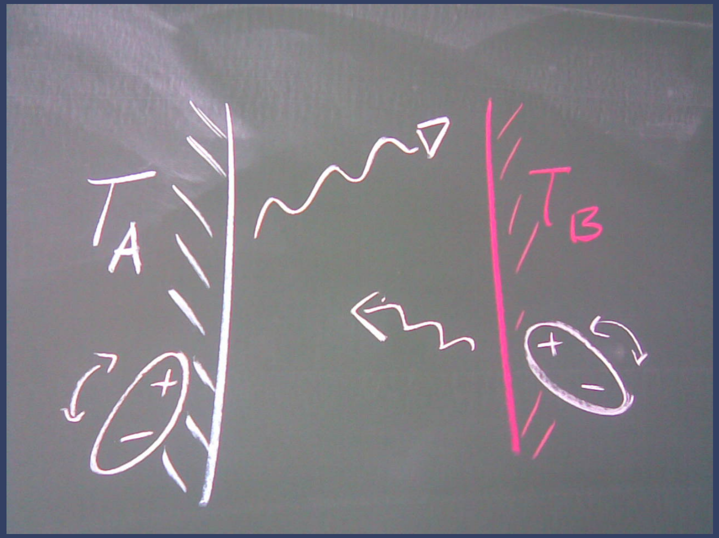
array of nano-particles
$\rightarrow$ P Ben-Abdallah

- core technique for non-equilibrium Rytov theory

  $=$ 'incoherent summation' over elementary source volumes

Examples

Radiative heat transfer



Poynting vector $\mathrm{Re}\,\langle \mathbf{E}^*(\mathbf{r},\omega) \times \mathbf{H}(\mathbf{r},\omega)\rangle_z$ T_{z0} current

$=$ (left sources) $+$ (right sources)

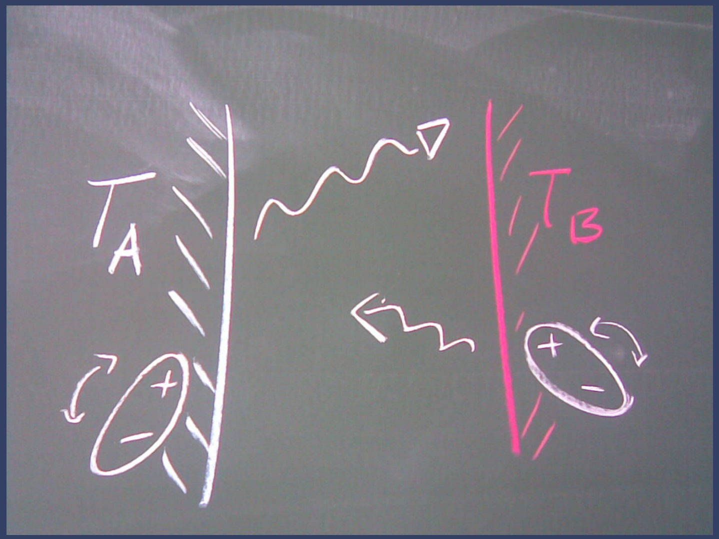
$\approx (T_B - T_A) \times h(\text{distance}, \bar{T})$

$\rightarrow$ this week's programme

- Casimir interaction $=$ momentum transfer, stress tensor $\langle T_{ij} \rangle$

Examples

Radiative heat transfer



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$= ($left sources$) + ($right sources$)$

$\approx (T_B - T_A) \times h(\text{distance}, \bar{T})$

$\rightarrow$ this week's programme

Typical / tacit assumptions

— homogeneous temperature profile *suggested problem* conduction faster than radiation

— Langevin sources not correlated radiative coupling slower than local relaxation

— no heat extracted from vacuum fluctuations but: unstable vacua

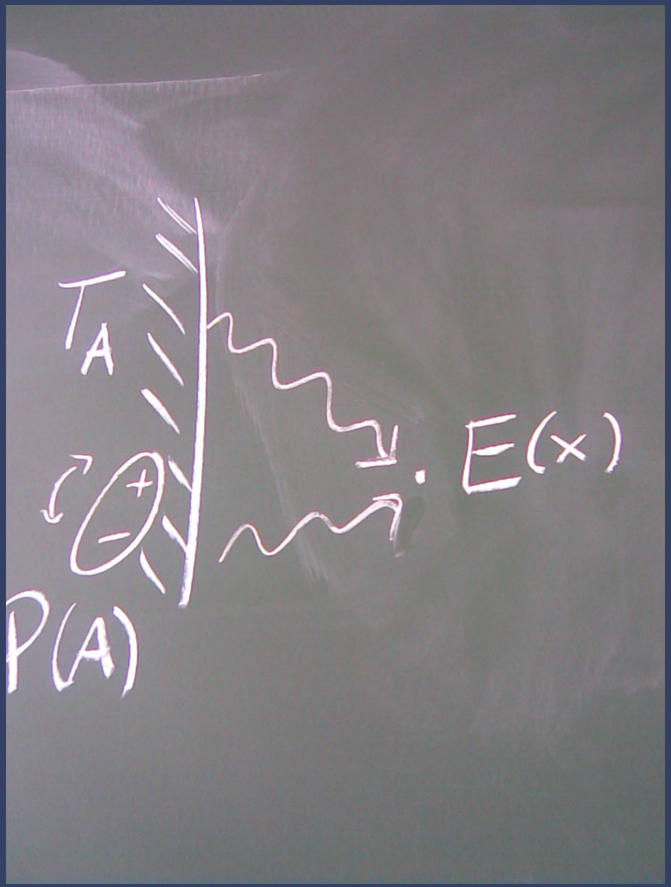
- Casimir interaction = momentum transfer, stress tensor $\langle T_{ij}\rangle$

Examples

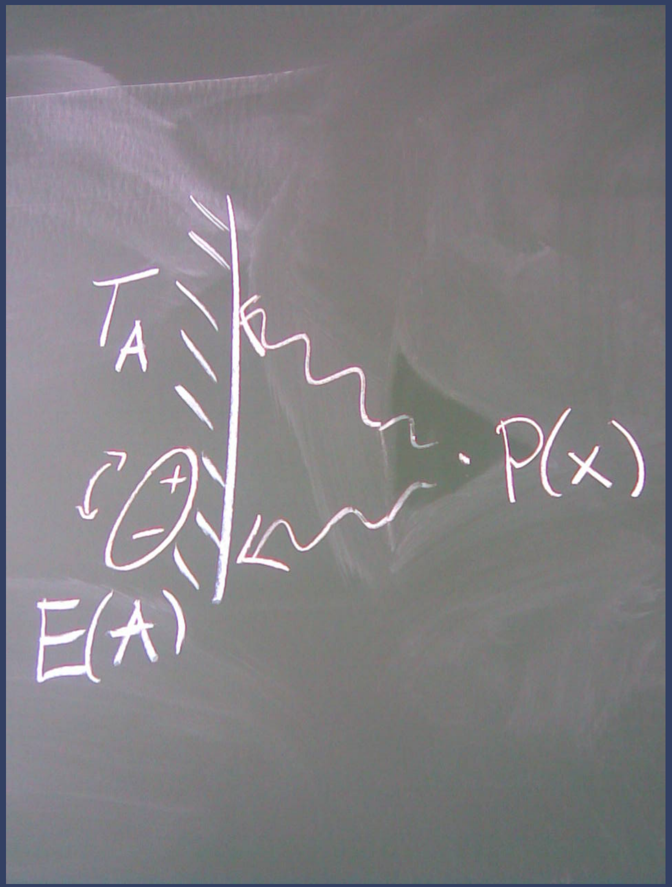
Radiative heat transfer

- convenient shortcut: generalized Kirchhoff law

Dorofeyev & Vinogradov (*Phys Rep* 2011)

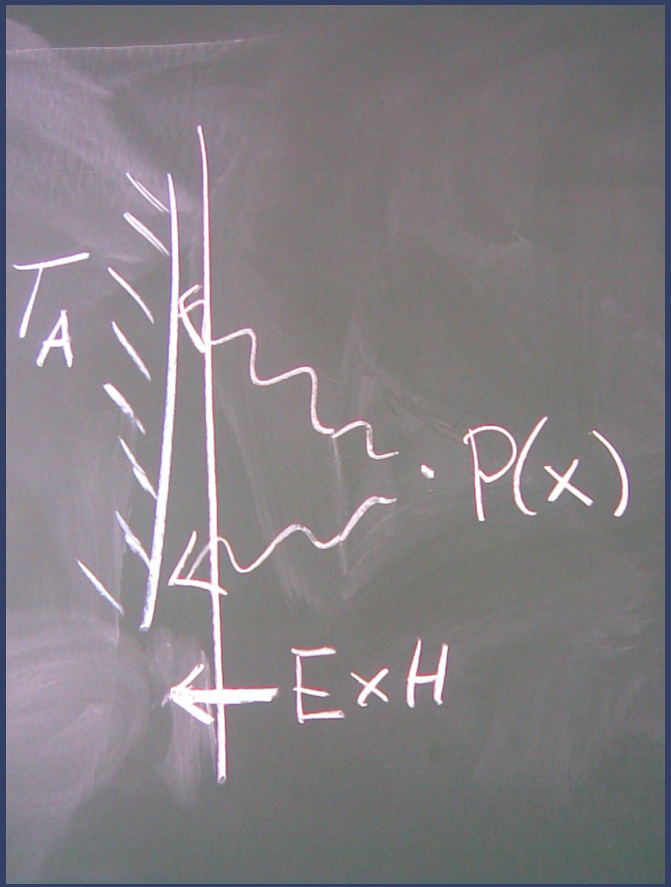


emission by sources



reciprocity

$$G(\mathbf{x},\mathbf{x}') = G^{\mathsf{T}}(\mathbf{x}',\mathbf{x})$$



Poynting vector

"Quantum" remarks

(1) 'first quantization'	QM	electrodynamics
Heisenberg	$\Delta p \Delta x \sim \hbar$	(Fourier)
Planck & Einstein	$\hbar\omega$	$\varepsilon \mathbf{E}^2 + \mu \mathbf{H}^2$
de Broglie	$\hbar \mathbf{k}$	$\varepsilon \mathbf{E} \times \mathbf{B}$
(2) 'second quantization'		
Pauli & Jordan		$\Delta E \Delta A \sim \hbar$

$$\mathrm{i}\,[A_i(\mathbf{x}), E_j(\mathbf{x}')] = \frac{\hbar}{\varepsilon_0} \delta^{\perp}_{ij}(\mathbf{x} - \mathbf{x}')$$

$$\mathrm{i}\,[B_i(\mathbf{x}), E_j(\mathbf{x}')] = -\frac{\hbar}{\varepsilon_0} \epsilon_{ijk} \partial_k \delta(\mathbf{x} - \mathbf{x}')$$

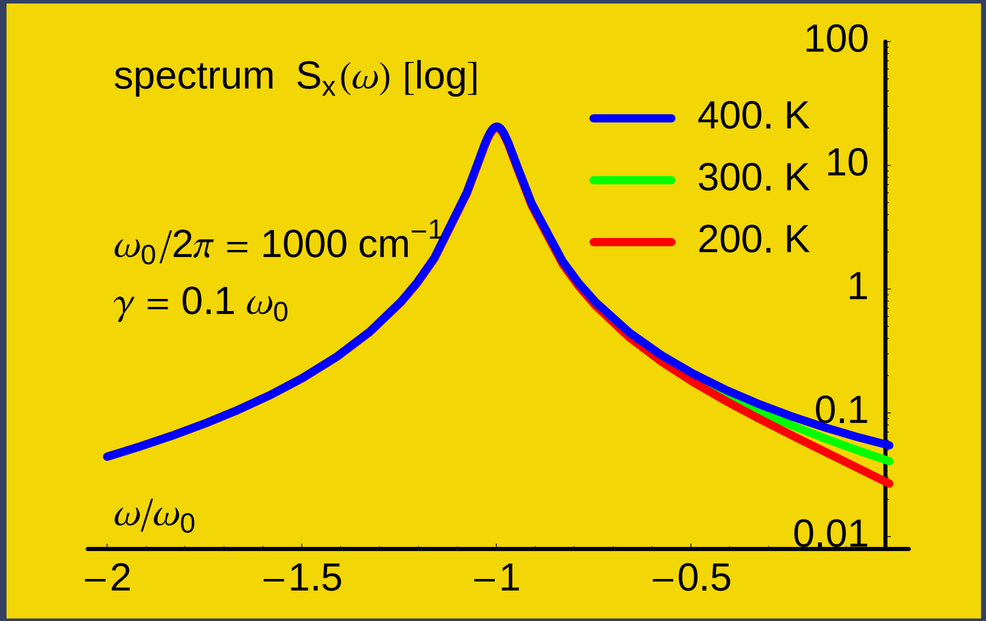
Ordering of operators

Feynman	$\langle T\{E(t)E(t')\}\rangle$	$D_E(\omega)$ even	
Green	$\mathrm{i}\langle[E(t),\, E(t')]\rangle\Theta(t - t')$	$\mathrm{Im}\, G(\omega)$ odd	
Hadamard	$\langle\{E(t), E(t')\}\rangle$	$H_E(\omega)$ real and even	$\sim \bar{N}(\omega) + \frac{1}{2}$
Glauber (?)	$\langle E(t)E(t')\rangle$	$S_E(\omega)$ positive	$\sim \bar{N}(\omega), \quad \omega > 0$
		$\mathrm{e}^{\mathrm{i}\omega(t-t')}$	

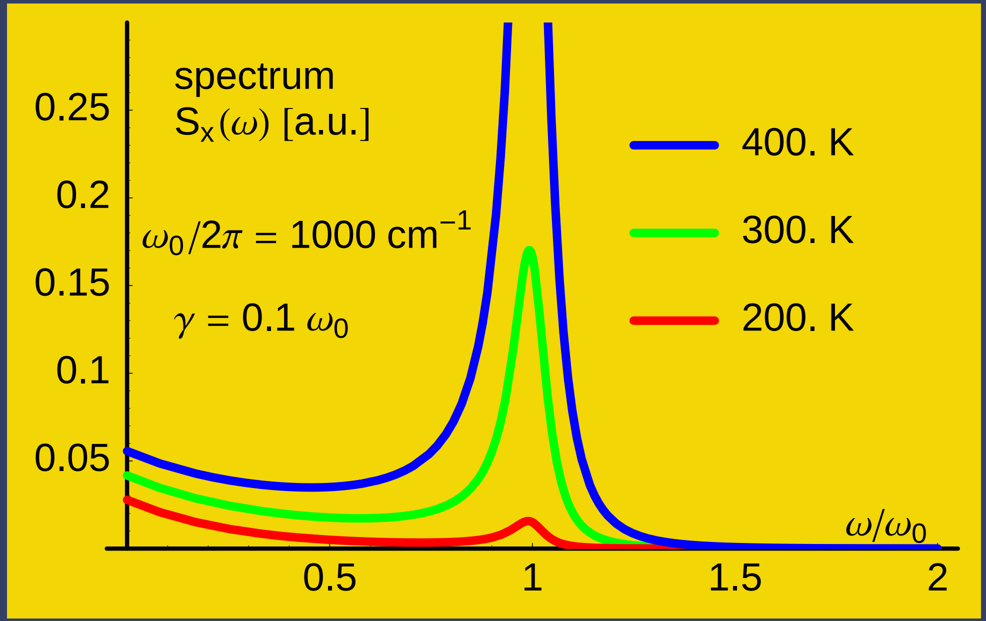
“Quantum” remarks

Damped oscillator in equilibrium

Glauber $S_x(\omega)$ at negative . . .



. . . and positive frequencies

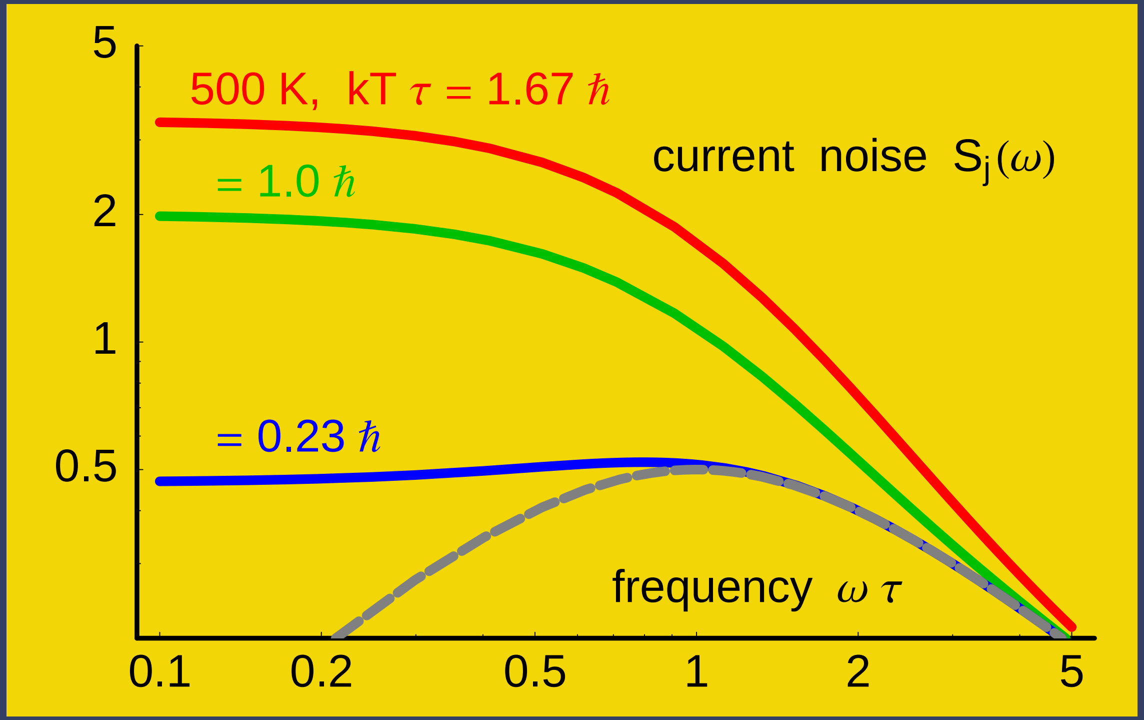


quantum noise / zero-point fluctuations

"Quantum" remarks

Electric current noise in metal

symmetrized $H_j(\omega) \sim (\bar{N}(\omega) + \frac{1}{2})\, \omega \, \mathrm{Re}\, \sigma(\omega)$



quantum noise of overdamped/diffusive field

- Casimir interaction: see Intravaia & CH (*Phys Rev Lett* 2009)

“Quantum” remarks

Suggested problems

Glauber spectrum: $-\bar{N}(-\omega) = \bar{N}(\omega) + 1$ asymmetric, still positive

Magnetic flux quantization? usually need electron charge for flux quantum, so it’s beyond field quantization? (cf. magnetic monopole)

Why do annihilation operators evolve with positive frequencies?

$$a(t) = a(0)\,\exp\left(-\mathrm{i}\omega t\right)$$

Can all observers agree on the sign of a frequency? (not always!)

anomalous Doppler effect (Ginzburg, review: *Phys Uspekhi* 1996):

$\omega' = \gamma(\omega + \mathbf{v}\cdot\mathbf{k}) < 0$

quantum friction, pair creation, Cherenkov radiation

Summary

stochastic electrodynamics Rytov = Maxwell + Langevin

route towards thermal equilibrium

QED: preserve commutators

separation of time scales

field absorption vs heat transport

Examples

stochastic field correlations: linear in sources

natural non-equilibrium technique (local vs global eq)

'macroscopic' response functions sufficient, even for quantum noise

(ε, μ, surface response ...)

Glimpses ... of the land beyond Rytov

Suggested problem

wikipedia entries about S. M. Rytov

(english, français, deutsch ...)

Thanks to the group!

Francesco Intravaia ($\rightarrow$ Nottingham)	Casimir physics	*Phys Rev A* 2010
Jürgen Schiefele ($\rightarrow$ Madrid)	atom chip & BEC	*Phys Lett A* 2011
Harald Haakh ($\rightarrow$ Erlangen)	superconducting atom chips	*Eur Phys J B* 2012
Geesche Boedecker (PhD)	non-Markovian QED	*Ann Phys (Berlin)* 2012
Giuseppe Cammarata (post doc)	quantization of coupled oscillators	
Alexander Kegeles (PhD)	entanglement production	
Gregor Pieplow (Dipl)	quantum friction	*New J Phys* 2013
Ralf Saplata (Dipl)	failures of adiabaticity	
Abdoulaye Diallo (Dipl)	border of BEC	

Timo Felbinger — Illarion Dorofeyev (Nizhny Novgorod)

Martin Wilkens — Vanik E. Mkrtchian (Ashtarak)

Appendix – forgotten references

von Laue: Thermal radiation in absorbing bodies (*Ann. Phys. (Leipzig)* 1910)

Bakker & Heller: 'Quantum' Brownian motion in electric resistances (*Physica* 1939)

Jauch & Watson: Phenomenological Quantum-Electrodynamics (*Phys Rev* 1948)

Callen & Welton: Irreversibility and generalized noise (*Phys Rev* 1951)

De Groot & al: Series on relativistic thermodynamics (*Physica* $\geq$ 1953)

V. L. Ginzburg: Electrical fluctuations (*Fortschr Phys* 1953)

Linder: Thermal Van der Waals interactions (*J Chem Phys* $\geq$ 1960)

van Kampen: FD relation in non-linear systems? (*Physica* 1960) vs Polevoi & Rytov (*Theor Math Phys* 1975)

Morris & Fürth: Spatial field correlations near conducting surfaces (*Physica* 1960)
see also Fuchs (*Radiophys Quant Electr* 1965)

Jaynes & Cummings: Quantum vs semiclassical radiation theories (*Proc. IEEE* 1963)

Agarwal: FD theorems and series on field quantization (*Z Phys* 1972; *Phys Rev A* 1975)

Boyer: Connection between Rytov and quantum electrodynamics (*Phys Rev D* 1975)

Eckhart: Problems with FD relations for heat transfer (*Opt Commun* $\geq$ 1982)

Henry & Kazarinov: Quantum noise in photonics (*Rev Mod Phys* 1996)

Appendix — Fluctuation-dissipation relation

Cross correlation spectrum:

$$\mathcal{E}_{ij}(\mathbf{x},\mathbf{x}';\omega) = \int\limits_{-\infty}^{+\infty} \mathrm{d}\tau\, \mathrm{e}^{-\mathrm{i}\omega\tau} \langle E_i(\mathbf{x}, t+\tau) E_j(\mathbf{x}', t)\rangle$$

Linear response:

$$\mathcal{G}_{ij}(\mathbf{x},\mathbf{x}';\omega) = \frac{\mathrm{i}}{\hbar} \int\limits_{0}^{+\infty} \mathrm{d}\tau\, \mathrm{e}^{\mathrm{i}\omega\tau} \langle [E_i(\mathbf{x},\tau),\, E_j(\mathbf{x}',0)]\rangle$$

Canonical ensemble: (KMS relation)

$$\int\limits_{-\infty}^{+\infty} \mathrm{d}\tau\, \mathrm{e}^{\mathrm{i}\omega\tau} \langle A(\tau) B(0)\rangle = \mathrm{e}^{\hbar\omega/k_B T} \int\limits_{-\infty}^{+\infty} \mathrm{d}\tau\, \mathrm{e}^{\mathrm{i}\omega\tau} \langle B(0) A(\tau)\rangle$$

$\rightarrow$ Fluctuation–dissipation relation:

$$\boxed{\mathcal{E}_{ij}(\mathbf{x}',\mathbf{x};\omega) = 2\hbar \bar{N}(\omega) \frac{\mathcal{G}_{ji}(\mathbf{x}',\mathbf{x};\omega) - [\mathcal{G}_{ij}(\mathbf{x},\mathbf{x}';\omega)]^*}{2\mathrm{i}}}$$

Reciprocity: $\mathcal{G}_{ji}(\mathbf{x}',\mathbf{x};\omega) = \mathcal{G}_{ij}(\mathbf{x},\mathbf{x}';\omega)$